Intermediate Microeconomics

Chapter 21: Cost Minimization

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The Cost-Minimization Problem

Consider a firm using two inputs to make one output.

The production function is

$$y = f(x_1, x_2)$$

Take the output level $y \geq 0$ as given.

Given the input prices w_1 and w_2, the cost of an input bundle (x_1, x_2) is $w_1 x_1 + w_2 x_2$

ISO-cost Curve

A curve that contains all of the input bundles that cost the same amount is an iso-cost curve.

Generally, given w_1 and w_2, the equation of the \$c iso-cost line is

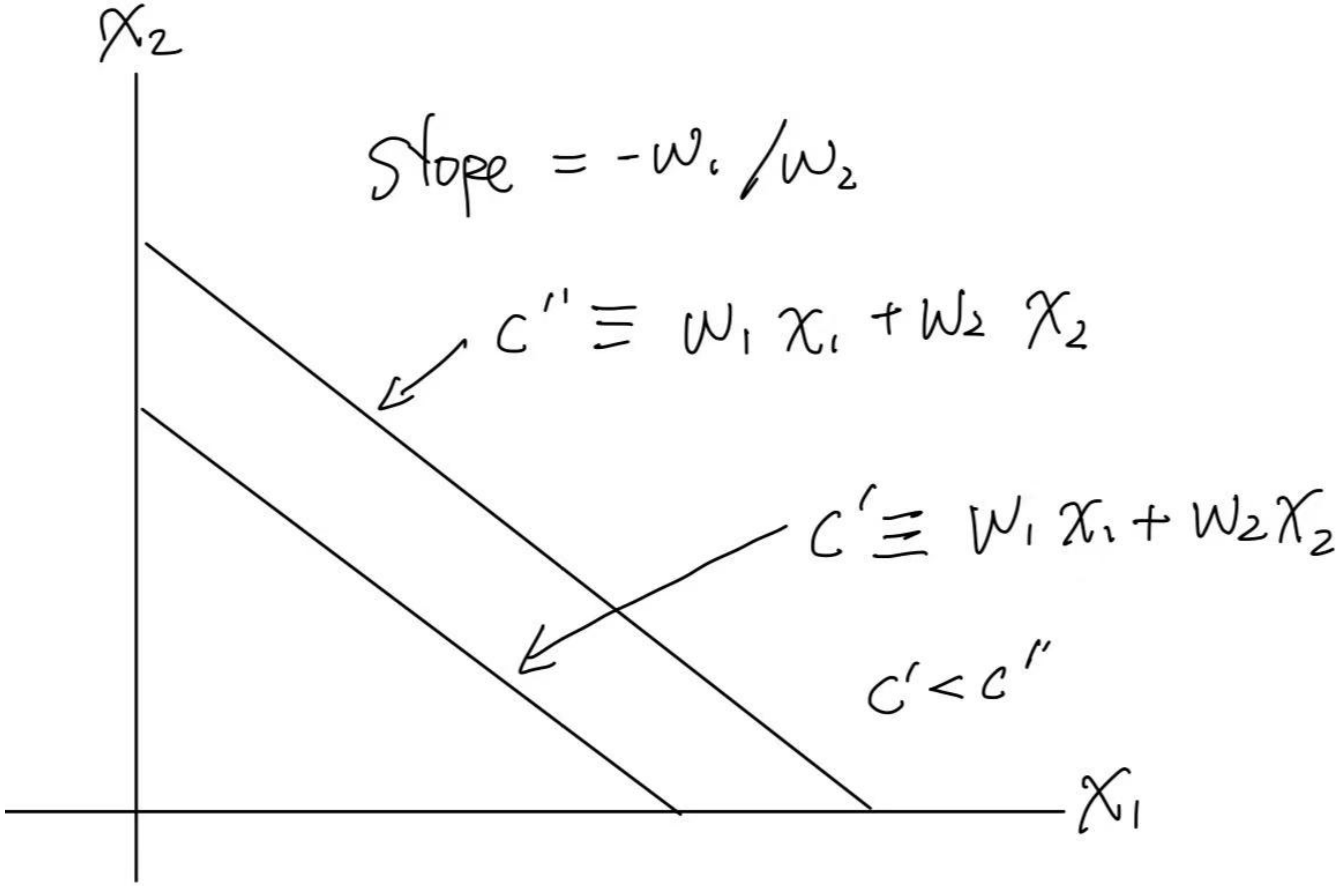
$$w_1 x_1 + w_2 x_2 = c$$

i.e.,

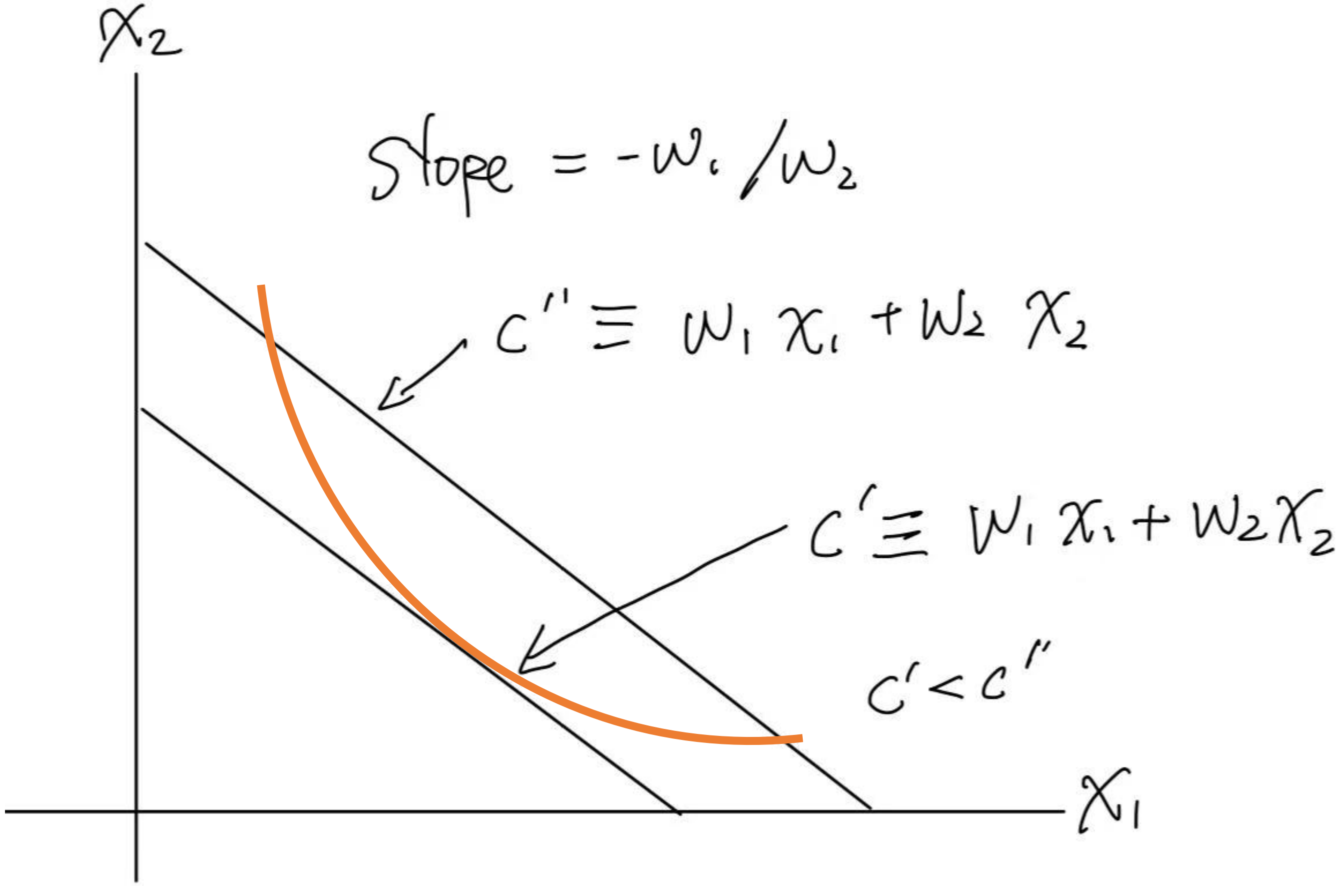
$$x_2 = -\frac{w_1}{w_2} x_1 + \frac{c}{w_2}$$

Slope is $-w_1/\, w_2$

ISO-cost Curve



Cost-Minimization



Cost-Minimization: An Exercise (Cobb-Douglas, CRS)

The production function is

$$y = x_1^{\alpha} x_2^{1-\alpha}$$

Prices: p, w_1, w_2. Solve the cost-minimization problem.

$$\min_{x_1, x_2} w_1 x_1 + w_2 x_2$$

$$s.t., \underset{y}{\cancel{f(x_1, x_2)}} = x_1^{\alpha} x_2^{1-\alpha} \underset{=}{\geq} \underline{q}$$

$y = q$

$$c(q) = \frac{w_1^{\alpha} w_2^{1-\alpha}}{(1-\alpha)^{1-\alpha} \alpha^{\alpha}} \cdot q.$$

$p.$

$\pi = p \cdot q - c(q)$

Cost-Minimization: An Exercise (Cobb-Douglas, DRS)

The production function is

$$y = x_1^{1/3} x_2^{1/3}$$

Prices: p, $w_1 = 1$, $w_2 = 2$. Solve the cost-minimization problem.

$$\min_{x_1, x_2} w_1 x_1 + w_2 x_2$$
$$s.t., f(x_1, x_2) = x_1^{1/3} x_2^{1/3} \geq \underline{q}$$

$\min\ x_1 + 2x_2$

s.t. $q = x_1^{\frac{1}{3}} x_2^{\frac{1}{3}}$

$\mathcal{L} = x_1 + 2x_2 - \lambda\left(x_1^{\frac{1}{3}} x_2^{\frac{1}{3}} - q\right)$

$$\text{FOCs: } \begin{cases} 1 = \frac{1}{3}\lambda x_1^{-\frac{2}{3}} x_2^{\frac{1}{3}} & ① \\ 2 = \frac{1}{3}\lambda x_1^{\frac{1}{3}} x_2^{-\frac{2}{3}} & ② \\ q = x_1^{\frac{1}{3}} x_2^{\frac{1}{3}} & ③ \end{cases}$$

$\frac{①}{②}$ $\frac{1}{2} = \frac{x_2}{x_1} \Rightarrow x_1 = 2x_2$ ④

③④ $\begin{cases} x_1 = 2^{\frac{1}{2}} q^{\frac{3}{2}} \\ x_2 = 2^{-\frac{1}{2}} q^{\frac{3}{2}} \end{cases}$

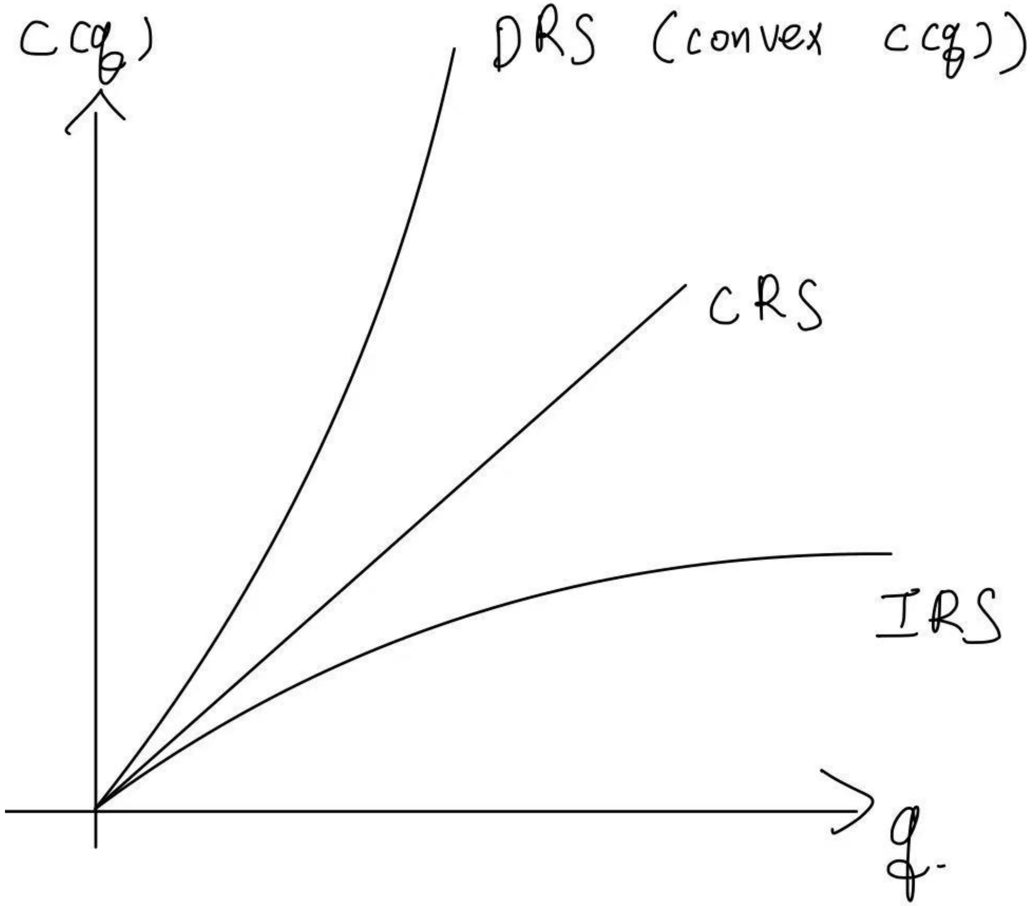
$c(q) = 2\sqrt{2}\, q^{\frac{3}{2}}$

Returns-to-Scale and Total Costs

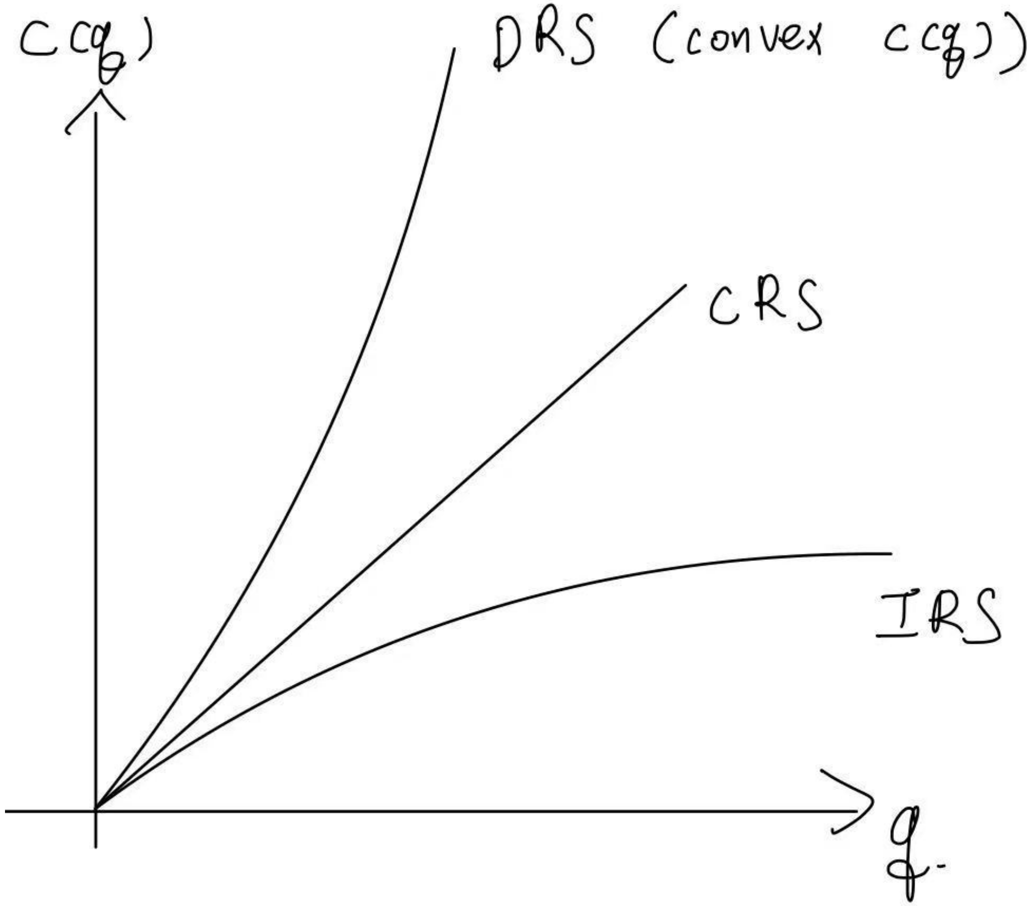
$$\pi(q) = pq - 2\sqrt{2}\, q^{\frac{3}{2}}$$

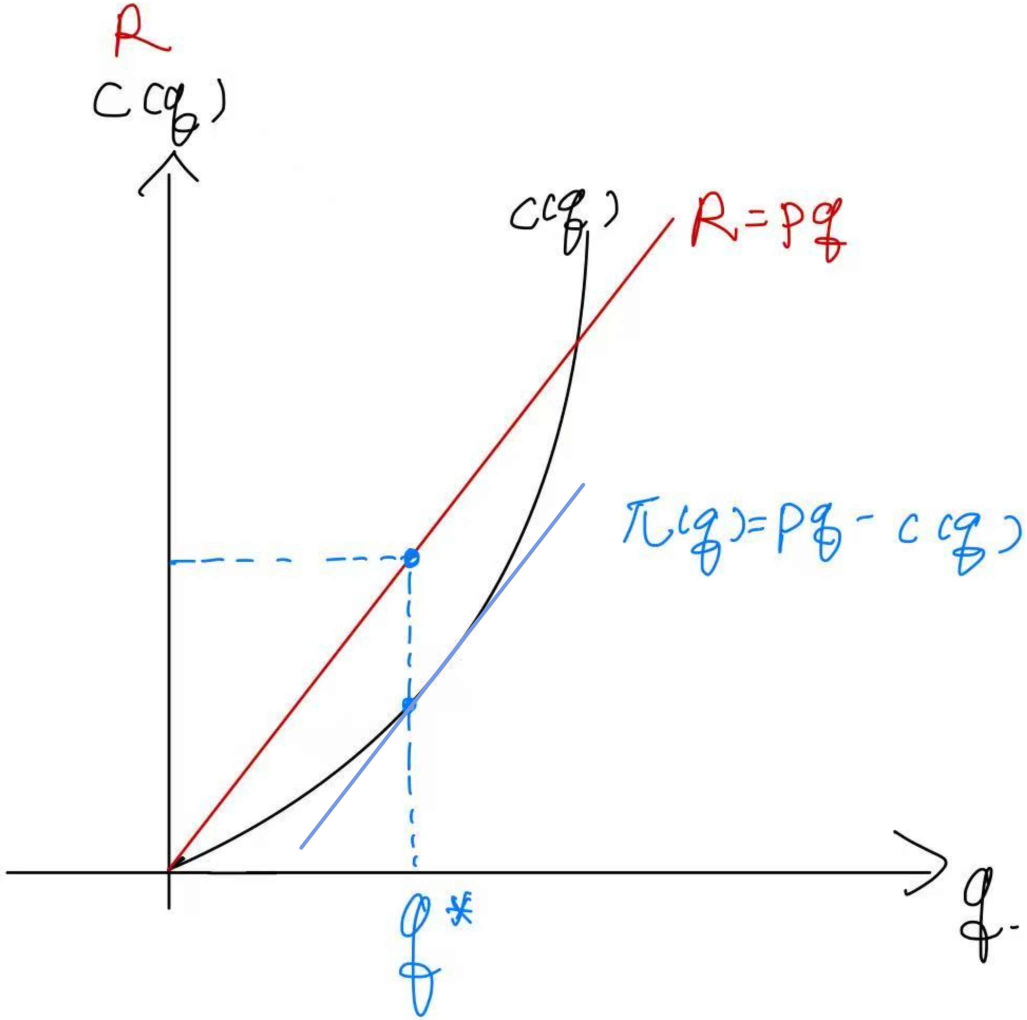
$$\pi'(q) = p - 3\sqrt{2}\, q^{\frac{1}{2}} = 0$$

$$q^* = \frac{p^2}{18}.$$



Returns-to-Scale and Total Costs

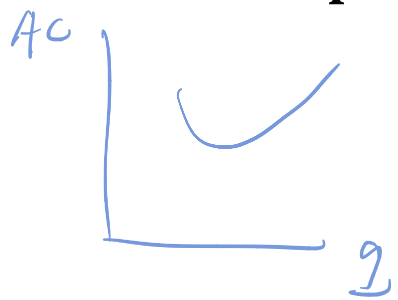




Returns-to-Scale and Av. Total Costs

For positive output levels y, a firm's average total cost of producing y units is

$$AC = C(q)/q$$

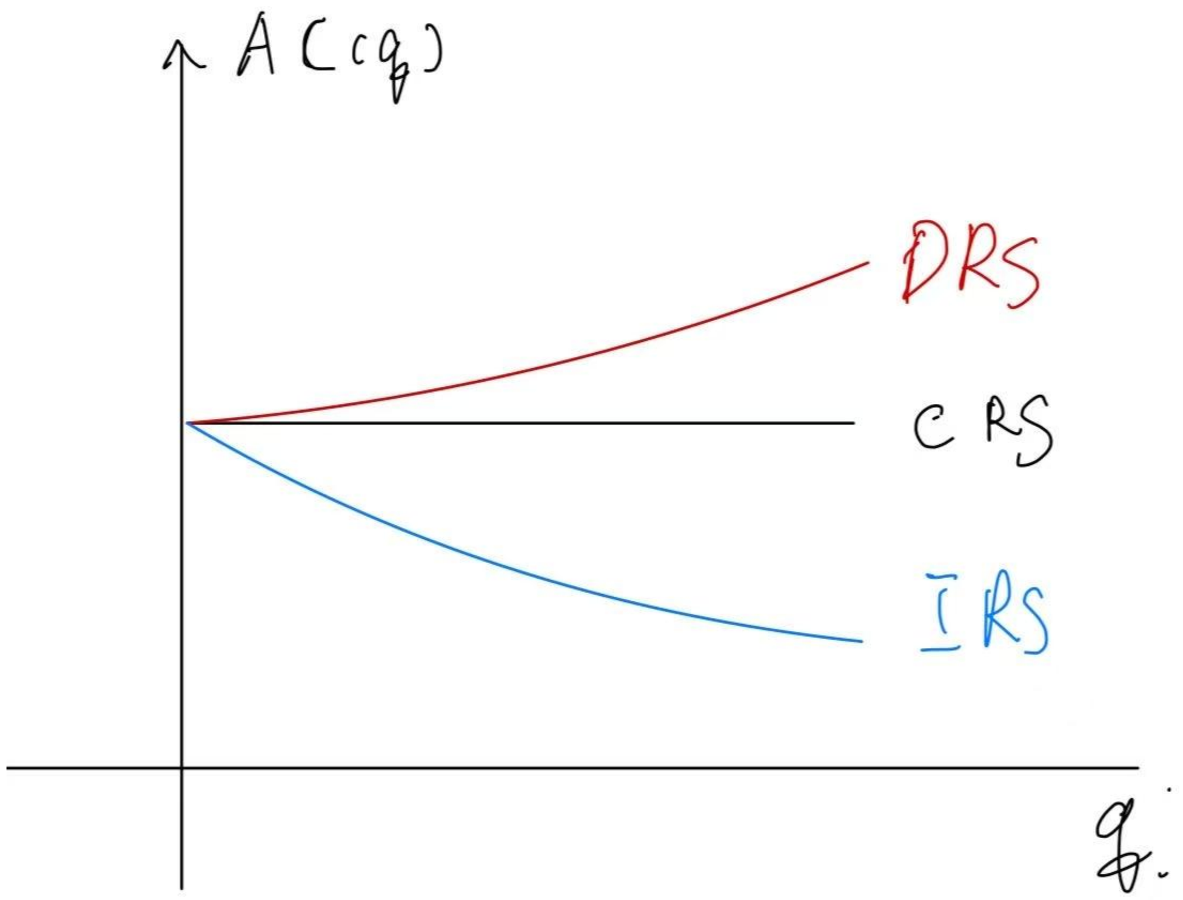


The returns-to-scale properties of a firm's technology determine how average production costs change with output level.

Our firm is presently producing y' output units.

How does the firm's average production cost change if it instead produces 2y' units of output?

Returns-to-Scale and Av. Total Costs



Cost-Minimization: An Exercise (Perfect Substitute)

The production function is

$$y = \alpha x_1 + \beta x_2$$

Prices: p, w_1, w_2. Solve the cost-minimization problem.

$$\min_{x_1, x_2} w_1 x_1 + w_2 x_2$$
$$s.t., f(x_1, x_2) = \alpha x_1 + \beta x_2 \geq \underline{q}$$

$\min w_1 x_1 + w_2 x_2$

$s.t.\ \alpha x_1 + \beta x_2 = \underline{q}$

$\Rightarrow w_1 x_1 + w_2 \frac{q - \alpha x_1}{\beta}$

$\underbrace{\left(w_1 - \frac{w_2}{\beta}\alpha\right)}\, x_1 + \frac{q}{\beta} w_2$

$\begin{cases} >0, & x_1 = 0,\ x_2 = \frac{q}{\beta} \\ <0 & x_1 = \frac{q}{\alpha},\ x_2 = 0. \\ =0, & \text{whatever} \end{cases}$

$x_1 = \frac{q}{\alpha} \quad x_2 = \frac{q}{\beta}$

$c_1 = q \cdot w_1 / \alpha \quad c_2 = q \cdot w_2 / \beta$

$c(q) = \min\{w_1/\alpha, w_2/\beta\} \cdot q.$

Cost-Minimization: An Exercise (Perfect Complement)

$\alpha x_1 < \beta x_2$.

$\min w_1 x_1 + w_2 x_2$,
s.t. $\alpha x_1 = \underline{q}$.

The production function is

$$y = \min(\alpha x_1, \beta x_2)$$

Prices: p, w_1, w_2. Solve the cost-minimization problem.

$$\min_{x_1, x_2} w_1 x_1 + w_2 x_2$$
$$s.t., f(x_1, x_2) = \min(\alpha x_1, \beta x_2) \geq \underline{q}$$

Intermediate Microeconomics

Chapter 22: Cost Curves

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Fixed, Variable & Total Cost Functions

F is the total cost to a firm of its short-run fixed inputs （固定投入）.

F, the firm’s fixed cost, does not vary with the firm’s output level.

VC is the total cost to a firm of its variable inputs （可变投入）when producing y output units. VC is the firm’s variable cost function.

VC depends upon the levels of the fixed inputs.

TC is the total cost of all inputs, fixed and variable, when producing y output units. TC is the firm’s total cost function;

TC=F+VC, or $c(y) = c_v(y) + F$

Types of Cost Curves

$TC = FC + VC$

$AVC = \frac{VC}{q}$

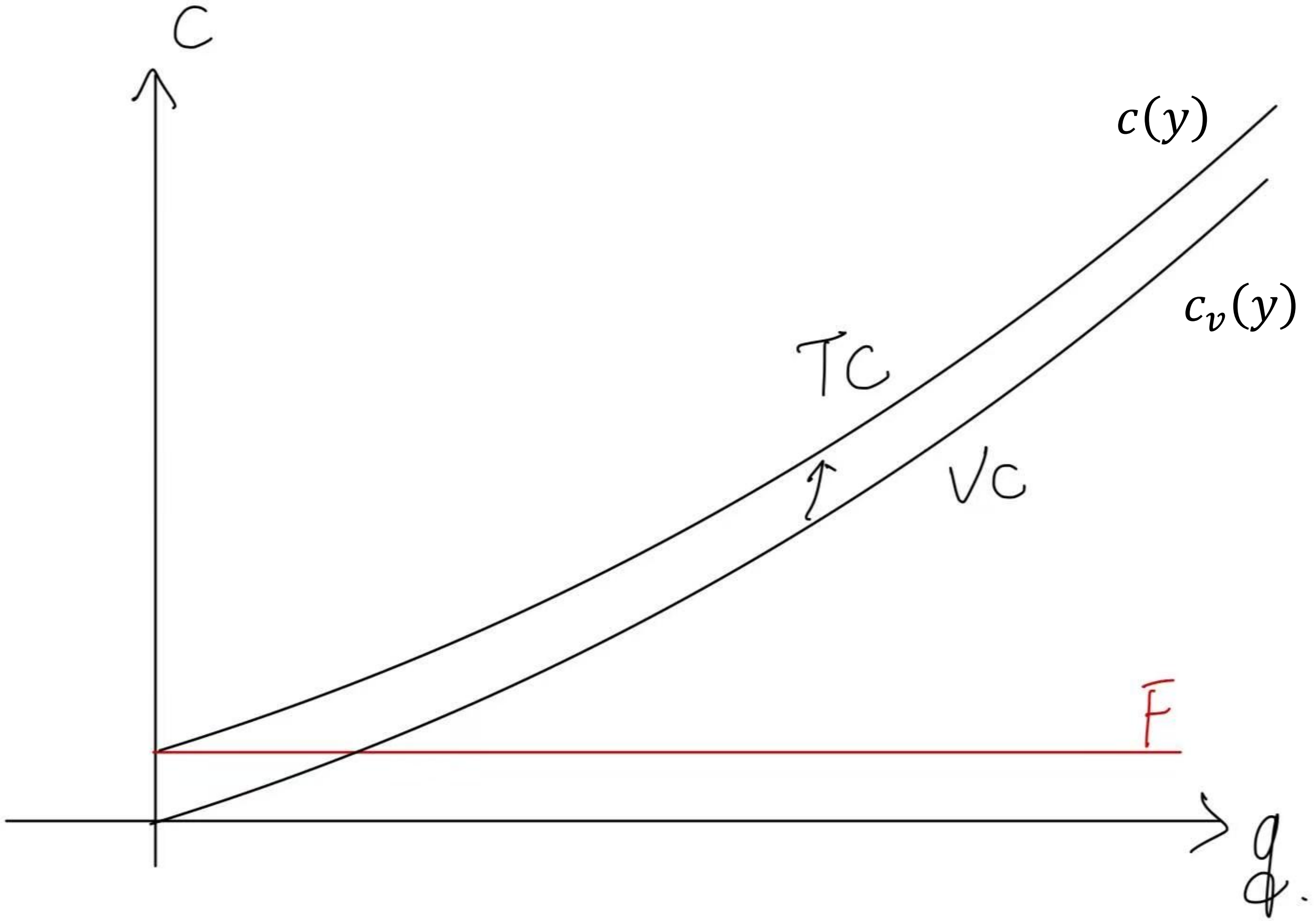
A total cost curve （总成本曲线）is the graph of a firm's total cost function.

A variable cost curve （可变成本曲线）is the graph of a firm's variable cost function.

An average total cost curve （平均成本曲线）is the graph of a firm's average total cost function.

How are these cost curves related to each other? How are a firm's long-run and short-run cost curves related?

Types of Cost Curves



Marginal Cost Function

Marginal cost is the rate-of-change of variable production cost as the output level changes. That is,

$$MC(y) = \frac{\partial c_v(y)}{\partial y}$$

The firm’s total cost function is

$$c(y) = c_v(y) + F$$

and the fixed cost F does not change with the output level y, so

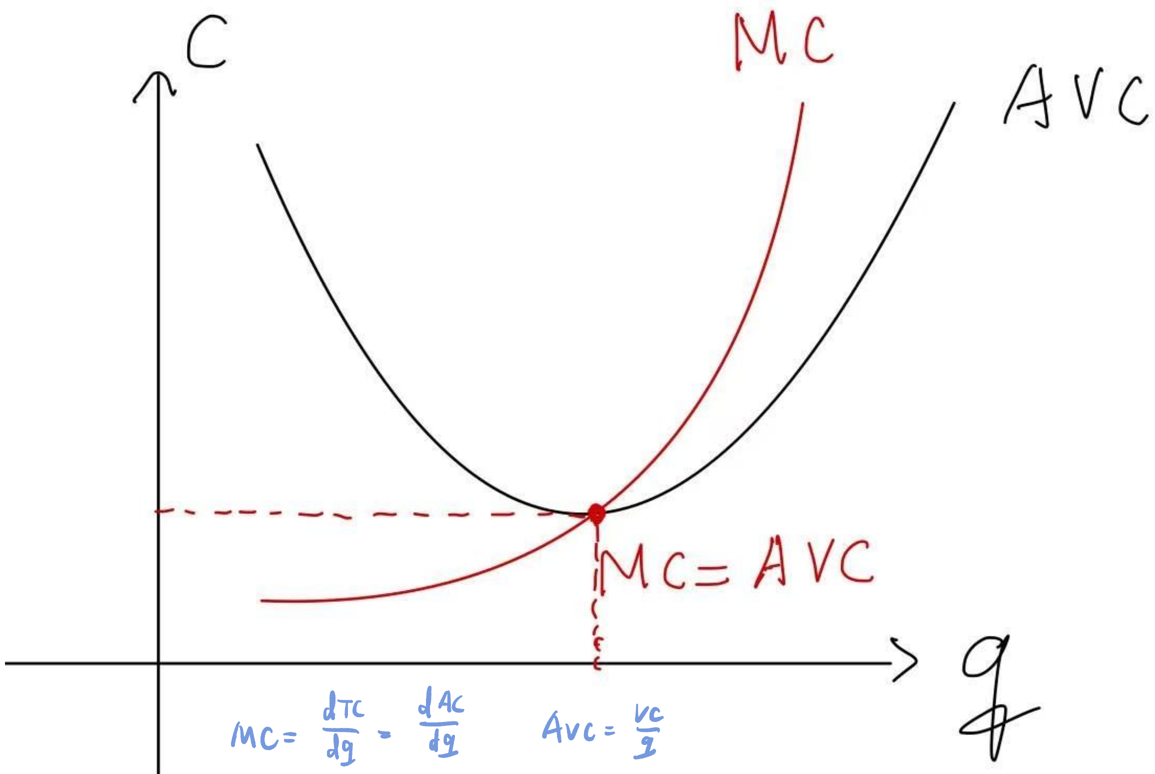
$$MC(y) = \frac{\partial c_v(y)}{\partial y} = \frac{\partial c(y)}{\partial y}$$

MC is the slope of both the variable cost and the total cost functions.

Marginal Cost and Returns to Scale

Q: When a firm operates under increasing returns to scale, is the marginal cost increasing or decreasing?

A: Marginal cost (MC) is decreasing under increasing returns to scale because production efficiency improves, reducing the cost per unit.



$$MC = \frac{dTC}{dq} = \frac{dAC}{dq} \qquad AVC = \frac{VC}{q}$$

$$\frac{dAVC}{dq} = \frac{dVC}{dq} \cdot \frac{1}{q} - VC \cdot \frac{1}{q^2} = \frac{1}{q}\left(\frac{dVC}{dq} - \frac{VC}{q}\right)$$

$$= \frac{1}{q}(MC - AVC)$$